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SURVEY

Multi-Shaker Random Vibration Control: What Is Published, and Where It Stops

Kevin Cross ORCID 0009-0002-6608-9426

Rests on **Published sources.** Published papers, reports, standards, a patent and controller makers' public pages, each read to the extent stated in its reference. The few numbers of its own are arithmetic and a Monte Carlo of the spectral estimate, shown in full and marked as the article's own; no new experiments or test simulations.

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Abstract

This survey maps what is published on controlling a random vibration test with several shakers, in which accelerometers on one test article are held to target spectra and, for every pair, a target coherence and phase. It covers the standards (MIL-STD-810H Method 527.2, IEC 60068-2-86, AECTP-400 Method 421), where targets come from and how their cross terms are chosen when the measured ones cannot be met, the closed-loop laws in print (Smallwood's factor correction and the matrix power law), identification and estimation, limits and notching, shaker placement and the points not controlled, structures that change with level, what controller makers and an open-source controller publish, and newer work. Each source is described as far as it was read. Within the sources read, the published work stops short of a tolerance for points not controlled, a comparison of closed-loop laws on one setup, any reported control of a structure that changes with level, and any learning-based controller with results. Two results are the article's own arithmetic: how far spectra, coherence and phase estimated from a finite number of averages scatter, set against the standard's tolerances, and how little the phases of a three-gauge target may disagree before it cannot be played. Nothing here was run on a shaker.

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Summary

A random vibration test with several shakers drives one test article from several points at once, so that a set of accelerometers on the article reads a target: a spectrum at each accelerometer and, for every pair of them, how closely their motions are tied and in what phase. This article maps what has been published on how such a test is specified, how the shaker drives are worked out before it starts and corrected while it runs, and where the published work stops. It draws on the standards, about fifty years of papers, the controller makers' public pages and the published documents of one open-source controller. A source is described only as far as it was read, and those that could not be had are named as such.

What the survey found:

- **A shared method.** Every source read that describes the start of a test measures the setup at a low level with uncorrelated random drives, inverts that measurement to get a first drive, and corrects the drive from loop to loop.
- **Two closed-loop laws in print.** One, Smallwood's correction of a factor of the target, is printed in full with its safeguards and code; a second, the matrix power law, is known here through a restatement. The standard describes other forms in a few lines, and the makers describe theirs by feature.
- **Several ways to choose the target's cross terms** when the measured ones cannot or should not be met: uncorrelated drives, the least drive, the article's own response to uncorrelated drive (the "buzz test"), and drives restricted to the patterns the setup responds to most.
- **Starting tolerances** in MIL-STD-810H Method 527.2: spectra within ±3 dB to 500 Hz and ±6 dB above, overall level within ±15 percent, and, where the coherence is 0.5 or more, coherence within ±0.1 and phase within ±10 degrees.
- **Placement studies** showing that a setup with as many shakers as control points can match the controls and still hold other points badly.

Where it stops:

- No standard read sets a tolerance for points that are not controlled.
- No comparison of different closed-loop laws on the same setup was found.
- No published test was found of control on a structure that changes with level, and no published rule for how far below full level to measure the setup.
- No learning-based controller with results was found.

Two pieces of arithmetic are this article's own. A controller that is exactly right still sees its measured spectra scatter: keeping all of 400 lines within ±1 dB nine times in ten takes about 260 averages, and a ±10 degree phase tolerance at a coherence of 0.5 is about the scatter of an estimate from 64 averages. And a target for three gauges can ask for the impossible: with a coherence of 0.9 on every pair, the three phases may disagree by only about 5 degrees before no test can meet it. Nothing here was run on a shaker.

1. What a Random Test With Several Shakers Is

1.1 Why Several Shakers

A random vibration test reproduces in a laboratory the broadband shaking a piece of hardware sees in service: on a launch vehicle, in an aircraft bay, on a truck. On one shaker the article is bolted to a fixture, an accelerometer near the fixing points is the control, and the controller raises or lowers the drive at each frequency until that accelerometer reads its target spectrum. The article is shaken in one direction at a time.

MIL-STD-810H gives the reasons for using several exciters instead (Method 527.2, 1.2.1): to spread the energy over a long or slender article "without relying on the dynamics of the materiel"; to supply more force to a

large article than one exciter can; and because several exciters allow “more degrees-of-freedom in accounting for both the impedance matches and the in service boundary conditions”. The published tests fall into two families. In the first, the article is mounted on a table driven by several exciters, often in all six rigid-body directions at once: a six-degree-of- freedom table. In the second, the article hangs free on soft supports, bare or on a fixture, and several shakers push on it at chosen points (Schultz and Avitabile 2020). That second family started with Daborn and co-authors, who reproduced a wind-tunnel environment on a freely supported article with small shakers (Daborn, Ind and Ewins 2014, known here from a secondary report of its abstract), in what Daborn named the **impedance-matched multi-axis test (IMMAT)** (Daborn 2017, title only). A related method, **intrinsic connection excitation (ICE)**, uses at least as many shakers and controls as the article has connection degrees of freedom. Zwink, Cross and Fowler (2023) say it “is similar to” IMMAT, except that “no attempt is made to match the impedance of the next level assembly and instead the vibration fixture is designed to be as light and stiff as possible”. In a twelve-shaker ICE test of an aerospace component, Larsen, Schultz and Zwink (2022) found “a much more accurate replication of system field test response compared with single degree of freedom testing”. Tuman and co-authors (2024) control only at a fixture that stands in for the next assembly, a transmission simulator.

Mayes and Rohe (2016) used six shakers in place of an acoustic test of an industrial structure (Section 3.1). Rohe, Schultz and Hunter (2021) describe a six-degree-of-freedom table with two modal shakers added, 14 drives controlling 21 accelerometers (Rohe 2022), where “Initial attempts using commercial vibration control software were unsuccessful” and the added shakers “improved the match to flight data over just the 6-DoF shaker”.

1.2 What the Controller Is Asked to Hold

A random vibration target is a **spectrum**: how much vibration there is in each narrow slice of frequency, in units such as g^2/Hz. The controller works on a grid of these slices, called **lines**, typically a fraction of a hertz to a few hertz apart; a group of neighbouring lines, such as a third of an octave, is a **band**. The accelerometers the controller reads and brings to their targets are the **control gauges**.

On one shaker the target is one spectrum. With several shakers and several control gauges it is more. Two gauges can each read their target spectrum while moving together, in opposition, or independently of each other, and the article is stressed differently in each case. Two more numbers per pair of gauges and per line describe how they move relative to each other. The **coherence**, from 0 to 1, is the share of the motion at one gauge that is tied, at that frequency, to the motion at the other. The **phase** says whether the tied part moves in step (0 degrees), in opposition (180 degrees) or somewhere between. The spectrum of one gauge is its **autospectrum**; the quantity that carries a pair’s coherence and phase is their **cross spectrum**.

All of these, for every gauge and every pair, make one square table per line, the **spectral density matrix** (Figure 1 b). Its diagonal holds the autospectra and the entries off the diagonal the cross spectra; the lower half mirrors the upper. The target of a multi-shaker random test is one such matrix per line, the **reference**. The drive to the shakers is described the same way: one matrix per line whose diagonal is each shaker’s drive spectrum and whose off-diagonal entries say how each pair of drives is correlated.

A spectral density matrix measured from real signals can never ask for negative power in any combination of its channels. A matrix with that property is called **positive semi-definite**. A reference without it cannot be met by any test, and a drive matrix without it cannot be played (Section 2.3).

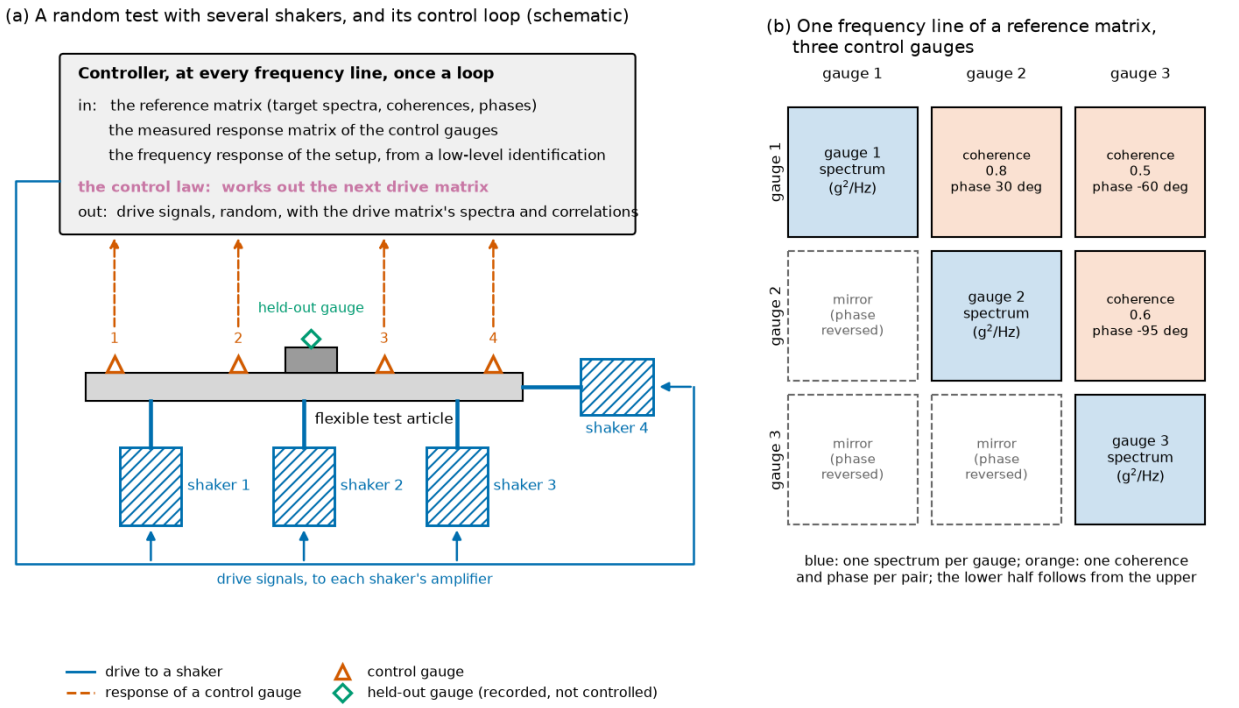


Figure 1: A random vibration test with several shakers (a) and one frequency line of the reference it is controlled to (b). Schematic only, not a particular test.

A **held-out gauge** is recorded but never shown to the controller; it is used to judge whether the article responded as it should away from the control points. A **rig** is the article on its supports or fixture with a given set of shakers and control gauges, as the controller sees it. These terms are the ones the first article in this journal uses (Cross and Zwink 2026).

1.3 The Loop

The controller's picture of the rig is its **frequency response**: on each line, a matrix H with one row per control gauge and one column per shaker, giving the acceleration at each gauge per volt of drive at each shaker. It is measured before the test, in a step called **identification**, by driving the shakers at a low level with random noise that is uncorrelated from shaker to shaker and recording the gauges. If the drive matrix is G_{uu}, the response matrix at the control gauges is

$$G_{yy} = H\, G_{uu}\, H^{H} \tag{1}$$

where H^{H} is the conjugate transpose of H (each entry's phase reversed, rows and columns swapped). A multi-shaker random controller has to choose G_{uu} on every line so that G_{yy} matches the reference, G_{rr}, as closely as the rig allows. The rule that chooses it is the **control law**. The controller then turns the drive matrix into random time signals with those spectra and correlations, plays them, estimates the response matrix from what the gauges read, and corrects the drive. One pass of play, estimate and correct is a **loop**.

The control literature writes a spectral density matrix as $G = E[XX^{H}]$, which Smallwood (1999) notes is the complex conjugate of the textbook cross spectrum; MIL-STD-810H prints both layouts (Method 527.2, Annex B 3.3). A phase moved from one convention to the other changes sign.

1.4 What a Rig Can Reach

On a rig with as many control gauges as shakers (a **square** rig) and an exact frequency response, Equation 1 can be inverted line by line and any playable reference can in principle be met. With more control gauges than shakers it cannot: four shakers cannot set the spectra, coherences and phases of twelve gauges independently. With fewer control gauges than shakers many drives meet the reference and one of them has to be chosen.

The tool used for all three cases is the **pseudo-inverse** H^{+} of the frequency response. The drive

$$G_{uu} = H^{+}\, G_{rr}\, H^{+H} \tag{2}$$

meets the reference exactly on a square rig, gives the response closest to it in the least-squares sense when there are more gauges than shakers, and gives the drive of least total power when there are fewer (Smallwood 1999; Smallwood 2005). Smallwood put the limit plainly: with more responses than drives, "it will not be possible to match more than n autospectra, where n is the rank" of the matrix, the number of independent patterns of motion the shakers can make. With more gauges than shakers, the response the rig can reach is the reference projected onto the responses the shakers can make; the distance between that and the reference is a property of the rig, not of the control law (Cross and Zwink 2026).

The pseudo-inverse is computed from the **singular value decomposition** of H on each line. In plain terms, it splits the rig into patterns of shaker drive, ordered from the one the gauges respond to most strongly to the one they barely feel, and gives how strongly they respond to each (the singular values). The ratio of the largest to the smallest is the line's **condition number**. A large one means that some pattern of drive barely moves the gauges, so a reference that asks for motion in that pattern takes a large drive, and a small error in the measured H becomes a large error in the drive. **Regularization** limits this by dropping or damping the weakest patterns before inverting (Section 4.5).

2. How a Random Test With Several Shakers Is Specified and Judged

2.1 What the Standards Say

Three standards have a method of their own for tests with several exciters. **MIL-STD-810H** (with Change 1, 2022) has Method 527.2, "Multi-Exciter Test". It names the configurations by how many exciters drive how many axes: several exciters in one axis (multi-exciter single-axis, **MESA**) and several exciters in more than one axis (multi-exciter multi-axis, **MEMA**), with the hexapod, the multi-axis shaker table and the cube named as common six-exciter examples and systems of "more than 6 exciters" as over-actuated ones (1.2.2). Its random procedure (Procedure II) extends the single-axis random method, 514.8, and the drives it synthesizes are stationary and Gaussian (2.2). It says what it leaves out: it "does not address aspects of vendor-supplied software control strategy" (1.3 a), and it "generally does not provide specific test tolerance information" (1.3 e). Its body says only that "The question of reasonable tolerances in a MET is not simple for either MET procedure" and that they "will often need to be addressed on a test by test basis" (4.2.2).

Its annexes do give numbers. Annex D 3 makes the single-axis autospectrum tolerances of Method 514.8 applicable to each control and allows the cross-spectral tolerances to be relaxed. Annex E 5.2 offers, as "a reasonable starting point", the only numbers in the method for a random test with several exciters (Table 1). The same paragraph warns that "the expected statistical variation imposes a lower limit on how tight the respective tolerance can be"; Section 5.2 puts numbers on that.

IEC 60068-2-86:2024, "Test Fx: Vibration, Multi-exciter and multi-axis method", covers MESA and MEMA with fixed-base shakers and with shakers attached directly to the article through flexible couplings, a setup "quite similar to that used for modal testing, but using environmental test severities" (clause 1). Unless specified

otherwise, the spectra of a random test “shall be within ±3 dB of the specified values” (6.2); clause 6 sets no tolerance on coherence or phase. Its annexes on control strategies and on choosing tolerances were not read for this survey (Section 12).

NATO AECTP-400 Method 421, “Multi-Exciter Vibration and Shock Testing” (read in its third edition, 2006), has three procedures (MESA, a multi-output procedure and MEMA) and says that “Specific multi-axis test severities are currently not defined in this test method due to the dependence of the test on the measured environment” (3). It sets abort limits of ±3 dB to 500 Hz and ±6 dB above, points to AECTP 401 (not read) for its tolerances, and where coherence and phase are specified says “it will be necessary to determine the optimum control limits from experiment” (5.2.4).

A recommended practice, IEST-RP-DTE022, “Multi-Shaker Test and Control” (2014), is meant to be used with Method 527; its contents were not available to this survey.

Table 1: Tolerances the standards give for a random test with several exciters, with the single-axis random tolerances of MIL-STD-810H for comparison. MIL-STD-810H with Change 1 (2022).

Source	Spectrum at each control	Overall level	Coherence and phase	Points not controlled
MIL-STD-810H Method 527.2, Annex E 5.2 (“a reasonable starting point”)	±3 dB to 500 Hz, ±6 dB above	rms of each controlled degree of freedom within ±15 %	coherence ±0.1 where it is 0.5 to under 1; phase: any where the coherence is under 0.5, otherwise ±10°, or ±40° near a steep change of the reference phase	none
MIL-STD-810H Method 514.8, Table 514.8-II (one axis, for comparison)	composite of the controls (average, maximum or minimum) ±3 dB to 500 Hz, ±6 dB above; each control under average control ±6 dB and ±9 dB	±10 % (average); ±25 % (each control)	not applicable	none
IEC 60068-2-86:2024, 6.2	±3 dB	none in clause 6	none in clause 6	none in clause 6
AECTP-400 Method 421 (Edition 3)	AECTP 401 5.1 (not read); abort ±3 dB to 500 Hz, ±6 dB above	AECTP 401 5.1 (not read)	“from experiment”	none

None of these sets a tolerance for a point that is not a control, although the response away from the controls is what most of the article lives with. Method 527.2 says that when the rigid-body motion is controlled, “The elastic modes are not controlled, since often the control authority over these modes does not exist” (Annex E 4.4 a). Hale and co-authors (2024), reproducing a 16-channel field matrix on an 1,800-pound pod with five shakers, wrote that applying classical single-axis tolerances to every channel and cross spectrum is “simply not feasible”. The first article in this journal proposes a practice for judging these tests, including at held-out gauges (Cross and Zwink 2026).

2.2 How the Reference Is Written

Smallwood (1999) defined the reference of a random test with several shakers by “the auto spectra of the m responses, the ordinary coherence between pairs of responses, and the phase relationship between pairs of responses”, because “Defining the reference in this manner removes the burden from the user of determining if the cross-spectrum is realizable and improves insight into the specification.” Method 527.2 says most commercial systems take the cross terms in this form, as relative phase and coherence (Annex D 2.1.1). It adds that a coherence of exactly 1 “results in a singular SDM” and that a value “slightly less than 1.0, (γ_ij = .95 to .98), greatly reduces the numerical concerns”.

Without measured cross terms, Method 527.2 says, “it almost always needs to be assumed that excitation environments are independent of one another (coherence values are near zero)” (Annex D 2.2.2). It calls independent references “typically a conservative approach” that “may be overly conservative near frequencies dominated by a single mode” (Annex E 4.6). It recommends, “If possible”, specifying the test “in terms of the rigid body motion” (Annex E 4.7 a). AECTP-400 Method 421 adds a further choice: where the cross terms measured in service and those of the laboratory setup differ, the test may take its cross terms from the laboratory setup (5.2.5). Section 3 follows that idea and the others the literature offers.

2.3 A Reference That Can Be Played

“Any specified spectral density matrix must be positive semi-definite to be physically realizable. In practice it must be positive definite” (Method 527.2, Annex E 4.5.2 a). A coherence between 0 and 1 on every pair is necessary for that and, for two gauges, enough. For three gauges or more it is not enough. Raising or lowering one gauge’s autospectrum does not change whether the matrix is positive semi-definite, so the coherences and phases alone decide it, and at high coherence the phases have to agree with each other: if gauge 1 leads gauge 2 by 30 degrees and gauge 2 leads gauge 3 by 30 degrees, a fully coherent reference must have gauge 1 leading gauge 3 by 60 degrees. Musella and co-authors (2018) print the condition for three channels and show that choosing each pair’s phase on its own “could result in the reference matrix being negative definite”. Figure 2 shows how much disagreement is allowed. With a coherence of 0.9 on every pair, a reference whose three phases miss closing the loop by more than about 5 degrees asks for negative power in some combination of the three gauges and cannot be played; at 0.7 the allowance is about 20 degrees, and at 0.5 about 45. These numbers are this article’s arithmetic from that condition.

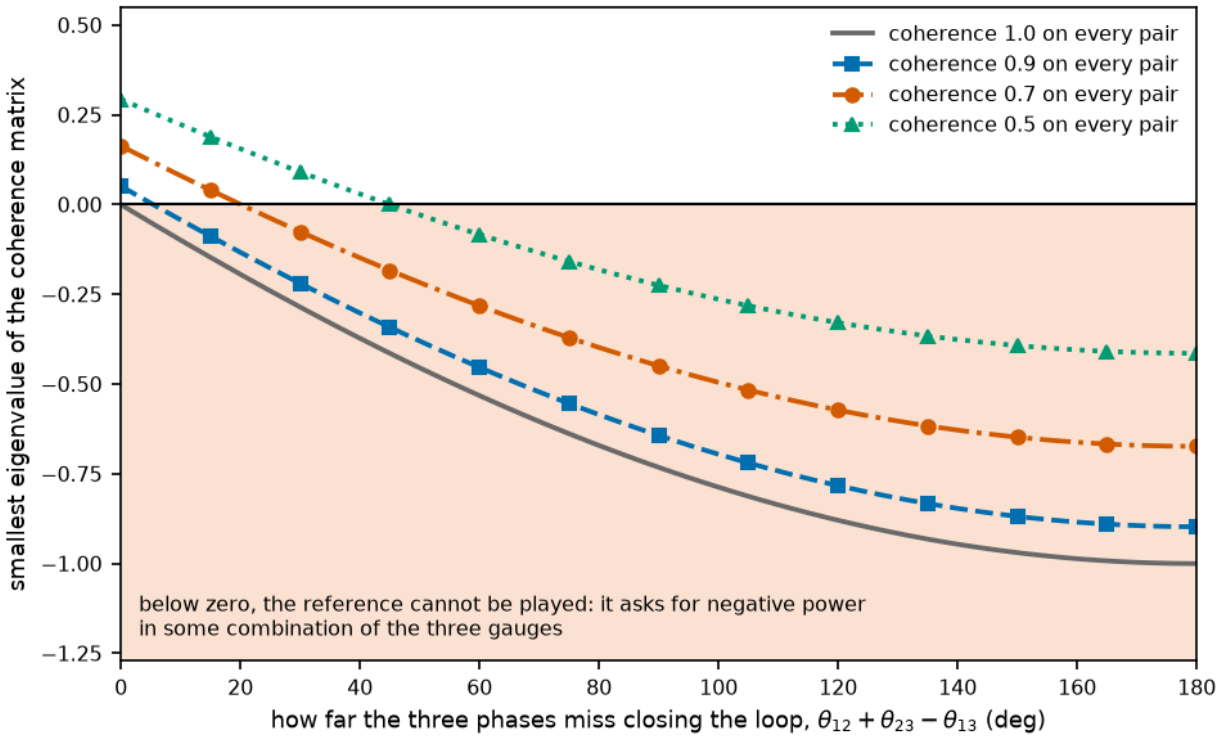


Figure 2: When a three-gauge reference can be played. Each curve is a reference with the same coherence on every pair and phases whose sum around the loop misses closing by the amount on the horizontal axis. Below zero the reference cannot be played.

A reference built pair by pair from separate estimates, or edited by hand, can fail this way without any single entry looking wrong. Hunter, Cross and Nelson (2018) note that a matrix computed from measured signals is positive semi-definite but that "field data are often noisy, modified, or enveloped. In these cases, it is easy to violate the positive definite requirement." Method 527.2 says the same of combining several field records: "Any type of enveloping operation should be avoided as it is highly likely to yield a non-positive definite result"; it averages the matrices instead, and where the result is still not positive definite it reduces every cross term's magnitude equally "until the Cholesky decomposition is possible" (Annex E 6, Steps 6 and 7). Even rounding can break a reference. Paripovic and Mayes (2020) report a controller that truncated every term at the sixth decimal place, which could turn a positive definite reference into one that was not, or push a coherence above 1; their workarounds were to round in advance while keeping the matrix positive definite, or to multiply the coherence by 0.9999, "which is undesirable".

3. Where the Reference Comes From

3.1 From Measurements in Service

When the motion was measured in service at the points that will be controlled, the reference is the measured matrix, processed as in Section 2.3. Often it was measured elsewhere: inside the article, or on a structure that is not part of the test. The drives that would reproduce it then have to be worked out through a model of the article, a step the literature calls **input derivation** or **input estimation**. Ross and co-authors (2017) compared four ways of deriving inputs for a six-degree-of-freedom shaker from flight measurements at ten internal gauges, among them a plain inverse and an iterative method with Tikhonov regularization, which damps every pattern of the inverse by a set amount (the Smallwood-Cap method), and found inputs in three translations

alone under-drove the responses; Jacobs and co-authors (2017) then ran the derived tests with no cross terms in the reference, with coherence only, and with coherence and phase. Mayes and Rohe (2016) worked out the voltages for six shakers to reproduce, at 37 internal accelerometers, an acoustic test of an industrial structure, from "a regularized inverse of the square of the amplitude of the frequency response function matrix and the power spectral density responses"; "No cross power spectral density responses are utilized." The sum of the 37 spectra followed the acoustic test's over most of the band to 4 kHz, less well above 3.5 kHz.

3.2 When the Rig Cannot Reach It

A reference measured in service may ask for motion the rig cannot make, or for cross terms that take far more drive than the autospectra do. Hunter, Cross and Nelson (2018) list six ways a test can treat the cross terms: let the structure set them; choose them to need the least drive; control them to the measured values (which works best where the controls move nearly independently); control the coherence but not the phase; force them near zero; or control autospectra at more points than there are drives and let the cross terms follow. The published methods are these.

Independent drives. Assume the drives are uncorrelated, solve for the drive spectra that give the reference autospectra, set any negative drive spectrum to zero, rescale the predicted response to the reference autospectra keeping its coherence and phase, and solve for the drive again. Method 527.2 prints these steps (Annex E 4.5.5.1) and credits them to a Smallwood paper at the Shock and Vibration Symposium of 2010; Schultz (2019) describes the same method from an undated Sandia report. Hunter, Cross and Nelson (2018) print a version they call the Nearly Independent Drive Algorithm, which "couples the drives 'just enough' to achieve the desired" autospectra; in their three-gauge example the resulting cross terms "have unity coherence". Their printed form works on vectors of spectra and ends in a drive spectrum rather than a matrix; the matrix form of the steps is Method 527.2′s.

Least drive. Smallwood (2005) found the cross terms that make the total drive power smallest for given autospectra: each pair's phase is set against the corresponding term of the inverse, and the inputs become fully coherent. He sets each pair's phase on its own and, in his worked example with more outputs than inputs, notes that "the method gives the minimum input that will control the trace of the outputs." With more than two outputs, setting the phases pair by pair can produce a reference that cannot be played (Section 2.3), and Method 527.2 notes that the technique "does not always guarantee the resulting SDM to be positive semi-definite" (Annex E 4.5.5). Musella and co-authors (2018) published that problem with a fix, setting some phases by the least-drive rule and computing the rest from the consistency condition, and tested it on a two-channel electronic simulator of a rig and on square, well-conditioned rigs of three and six drives. D'Elia and co-authors (2020) review techniques for reducing drive power and add one for the case where one drive takes most of it (read as an abstract). Method 527.2 says that "general techniques to address minimum drive criteria have not been formally established at the time of this publication" (Annex E 4.5.5.1 a).

Cross terms from the rig (the buzz test). Drive every shaker with uncorrelated noise with the article in its test setup, and take the coherence and phase of the response as the reference's cross terms, keeping the specified autospectra. The idea is Daborn's (Daborn 2014, his thesis, not read for this survey); Schultz (2019) and Schultz and Avitabile (2020) describe it and credit him. On a simulated plate with nine inputs, ten control gauges and two held-out gauges, under an acoustic field as the truth, Schultz found the buzz test "had the best balance of response accuracy and low input forces in this case"; the full-matrix inverse matched the control spectra but needed "much more force" and had "quite a lot of error" at a held-out gauge, and "Phase is not captured well by any of the methods." He concluded that comparing spectra at the control gauges alone "is not a sufficient metric". AECTP-400 Method 421 allows the same choice (5.2.5), and one controller maker describes it as its practice (Section 9).

A constraint on the drive shapes. Schultz and Avitabile (2020) restrict the drive to the few patterns the rig responds to most strongly at each line (the leading right singular vectors of H), calling it "an extreme form of regularization". The number of patterns is set at each line by how many singular values, divided by the largest, exceed a threshold "chosen by the test engineer"; they show thresholds of 0.05 to 0.5. On a simulated plate with four shakers the mean rms force fell 46 percent against the full inverse, and with eight shakers 80 percent, so that "a test using shape-constrained input estimation could be run at 12 dB higher levels … assuming that input force is the main limiting factor". On a plate in an acoustic field with three shakers the rms input fell 10, 33 and 50 percent. With one pattern, every response is fully coherent with every other.

Pareto targets. Coletti, Davis and Schultz (2026) derive open-loop inputs from the diagonal of the reference by weighing input energy against control error, giving a set of best compromises rather than one answer. They also point out that a dB error "tends to infinity" as a response goes to zero, so that a dB metric "may be significantly skewed", where a residual of the matrix trace stays bounded.

4. The Published Control Laws

4.1 The Open-Loop Solve and Where It Came From

Every published law starts from Equation 2: measure H, invert it, and compute the drive that would give the reference if H were exact. Smallwood (1999) records the lineage. The first random algorithms (Fisher 1973; Smallwood 1978; Smallwood, Woodall and Buksa 1986) needed a square H; Smallwood (1982) made "some provision" for one that is rank deficient; Paez, Smallwood and Buksa (1987) made "some effort" to control more points than drives; and "the use of the Moore-Penrose pseudo inverse is superior." The 1982 paper's abstract says its algorithm "corrects both the auto spectra and the cross spectra of the control points in closed loop". None of these earlier papers was obtained for this survey. Smallwood also notes that since 1978 at least four companies had sold such controllers and that they "rarely discuss the technical aspects of their systems in detail for competitive reasons."

Underwood and Keller (2001), writing from the controller-maker side, give the same start: the initial drive is $H^{-1}RH^{-H}$, with H^{-1} called the "system impedance matrix". Because H can be close to singular, the ratio of its smallest to largest singular value is compared with a user's "singularity threshold", and below it a pseudo-inverse is used, which "mitigates the tendency for the actuators to 'fight' and also limits the magnitude of the drives". On a plate of about two feet square on four shakers they show the impedance matrix singular from about 6 Hz to almost 40 Hz. Their closed loop is "a proprietary MIMO-random control method", and they do not print it.

4.2 Correcting a Factor of the Reference

The one closed-loop law printed in full, with its safeguards and code, is Smallwood's (1999). It does not correct the drive matrix directly. It factors the reference as $G_{rr} = RR^H$ with R lower triangular (a Cholesky factor "with zeros", after Dodds and Robson 1975 as Smallwood cites them, so that a reference that is only semi-definite can still be factored), plays independent random sources through a coupling matrix Z that starts as R, and through the inverse $G = H^+$ to the shakers. The drive matrix is then $H^+ZZ^HH^{+H}$, which is positive semi-definite whatever Z is: a drive built this way can always be played. After each loop the error between the reference and the measured response, $E = G_{rr} - G_{yy}$, gives a lower-triangular correction Δ from

$$\Delta Z^H + Z\Delta^H = E, \qquad Z_{new} = Z + e\,\Delta \tag{3}$$

solved column by column, with a convergence factor e (Smallwood 1999, Eqs. 11 and 12). The rules around it carry most of its practical knowledge. The autospectra are corrected first, in order of the channels' importance;

a cross term "will be increased only if the affected power spectral densities are not above their respective references"; and when they are above, a second form corrects the size of the term but not its phase. The appendix code limits each diagonal correction to half the larger of the term and the first diagonal, keeps diagonal terms from going to zero or negative, stops off-diagonal terms from sticking at zero, and keeps the number of nonzero rows and columns of Z no larger than the rank of H, "as determined by a tolerance". The paper's worked example is a rigid rig of three shakers whose H has rank one, asked for three independent responses: after 60 loops at $e = 1$ the autospectra match and the drive is shared equally, but "the cross-spectra will be much different than originally desired. We cannot make the responses independent." (The printed Eq. 12 divides one term by the row's diagonal where the paper's own code and the algebra use the column's; the code is the reliable statement.)

Manring and co-authors (2023) give the name Difference Control Algorithm to Smallwood's 1982 correction, which they restate in the same additive form, and note "a potential numerical issue where it is possible for the algorithm to require the shaker to output negative power"; their 2021 slides make the same point about the correction step (Manring, Mann and Schultze 2021).

4.3 The Matrix Power Law

Cui, Chen, He and Jiang (2011) proposed a matrix power control algorithm (MPCA). Their paper was not obtained for this survey; its update is known here as Manring and co-authors (2023) restate it for a square rig. With L_d a factor of the reference and $L_{s,k}$ a factor of the response measured at loop k,

$$E_k = L_d \, L_{s,k}^{-1}, \qquad L_{k+1} = E_k^{\varepsilon} \, L_k, \qquad 0 < \varepsilon \leq 1 \tag{4}$$

and the drive is $H^{-1} L_k$ times a matrix of random phases. The correction multiplies rather than adds, and an exponent ε sets how much of it is taken each loop. Manring and co-authors credit it with removing the negative-power problem "while maintaining the ability to control the full CPSD matrix", and their slides attribute to the same paper a simpler law that corrects the autospectra only.

Manring and co-authors (2023) then worked on how fast it converges, which they say "has received little attention". Smoothing the measured response matrix over neighbouring lines lets each loop use fewer frames, and they tuned the exponent and the smoothing in simulation by Bayesian optimization ("All optimization was done in simulation, as it would be infeasible to perform this type of optimization in experiment due to needing to run hundreds of tests"). They report convergence "within 12 drive-frames" on a simulated fixed-fixed beam with two base inputs and two outputs, and ran it on a free-free box-and-plate assembly with two shakers through their own controller. A constant exponent of 0.25 did as well as a scheduled one.

Later papers by the same authors and their colleagues at Nanjing University of Aeronautics and Astronautics, read here only as titles or abstracts, extend multi-shaker random control: a Tikhonov filter for noise (Cui and co-authors 2016), a time-domain approach (Cui, Chen and He 2017), a limit strategy (Zheng and co-authors 2023), references with set kurtosis and non-stationary references, and a correction that handles the autospectra and the cross terms with two separate factors, whose abstract states that with more than two control channels "the coherence coefficient and phase difference in the cross spectral densities cannot be controlled independently" under the existing matrix power correction (Zheng and co-authors 2024).

4.4 What the Standard Says About the Update

MIL-STD-810H does not give an algorithm ("the various MET control system vendors do not always approach control in the same manner", Annex B 3), but describes the update in a few lines: the drive is $D = Z \hat{R} Z^H$, with the impedance Z updated as new information comes in or a modified reference $\hat{R}$ computed from the error; or an additive drive error $D_e = Z(R - C)Z^H$ is computed; "Sometimes an adaptive correction is used.

Sometimes a combination of all methods is used" (Annex E 3.2 a). It names two families, updating the transfer function every loop (which is not possible at "any frequencies for which the drive signals are fully correlated") and correcting the coupling matrix from the error, where "Typically, some fraction of the error is applied" (Annex B 3.5). It records a change in priorities: earlier algorithms corrected the autospectra first and the cross terms "to the degree possible without corrupting the ASD terms", while "In modern MET algorithms, the drive signals are updated such that the SDM matrix has minimal mean-squared error" (Annex D 2.2). It gives no gain, no rule for keeping the additive form positive semi-definite, and no comparison of the forms.

4.5 More Controls Than Shakers, Transformations and Regularization

With more control gauges than shakers, older practice split the responses into "primary" responses, as many as the drives, and "secondary" ones (Smallwood 1999). Method 527.2 calls the square case "the most common control method", says that with more control channels than exciters the pseudo-inverse gives a least-squares answer and with fewer "exact control of the control SDM is often not possible. In such cases, some kind of average control is usually implemented", and that transformations are often used "to force square control" (Annex E 3.2 d). A **transformation** is a fixed matrix that turns the measured accelerations into the quantities controlled (for example the six rigid-body motions of a table from twelve accelerometers) or a few control signals into the drives of many shakers (Annex E 4.4); Method 527.2 builds it from the gauges' positions and directions and recommends more control channels than rigid-body degrees of freedom (Annex A 2.2). Underwood and Keller wrote on rectangular control (2004, read as an abstract) and on coordinate transformations (2003; Method 527.2 cites it as a Sound and Vibration article of 2006), the latter not obtained here.

Regularization is the published answer to a poorly conditioned H. Schultz (2019) compared truncating the small singular values, raising them, and Tikhonov regularization with a value scaled on each line. On a simulated plate with nine inputs and twelve outputs, input estimation errors "seem to correspond to condition numbers above around 1000"; too little regularization does not help and too much changes the form of the matrix. Placement mattered more than count: "It is not simply the number of gages which determines the condition number." Schultz and Rohe (2026) give the trade in one sentence: "more regularization results in poorer response accuracy at the control locations but lower input levels and potentially more reasonable results at uncontrolled locations", so "using regularization of some kind is generally recommended." In practice, Paripovic and Mayes (2020) ran a twelve-control test on a six-degree-of-freedom table with a condition-number limit of 1,000, and with a higher limit "the controller could not run the environment"; Tuman and co-authors (2024) found their simulations "consistently more predictive" of their tests once they added the condition-number threshold the commercial controller applies.

Table 2: The closed-loop laws for multi-shaker random control found in the published record, and how much of each is printed.

Law	What it corrects each loop	Model used	Where printed	Shown on
Open-loop pseudo-inverse	nothing	H from identification	Smallwood 1999; Underwood and Keller 2001; MIL-STD-810H 527.2 Annex B	the starting drive of every law below
Factor correction (Smallwood's; Manring and co-authors call the 1982	a factor of the reference, by addition; autospectra first; a second form for size only	H^{+}, fixed or updated	Smallwood 1999, Eqs. 11 to 13 and code; Smallwood 1982 (abstract)	a rigid three-shaker rig of rank one (printed numbers)

Law	What it corrects each loop	Model used	Where printed	Shown on
form the Difference Control Algorithm)				
Matrix power (MPCA)	a factor of the reference, by multiplication, with an exponent	H^{-1}, square rig	Cui and co-authors 2011, as restated by Manring and co-authors 2023	a simulated beam and a box-and-plate assembly, two shakers
Autospectra-only ratio	the diagonal only	H^{-1}	attributed to Cui and co-authors 2011 in Manring, Mann and Schultze 2021	not stated
Additive drive error	the drive, by $Z(R - C)Z^H$	$Z = H^+$	MIL-STD-810H 527.2 Annex E 3.2 a	not stated
Separate auto and cross corrections	two factors, one for each	not stated in the abstract	Zheng and co-authors 2024 (abstract)	simulation (abstract)
Adaptive impedance update	the impedance matrix every loop	Z updated	Underwood and Keller 2001 (not printed; "proprietary")	two shakers; a plate on four shakers

5. Identification, Averages and the Drive Signals

5.1 Measuring the Rig

The sources read that describe it measure H the same way: drive every shaker at a low level with random signals that are uncorrelated from shaker to shaker, and fit H from the drive and response spectra; Smallwood (1999) notes that driving one shaker at a time also works. The drives must be at least partly uncorrelated because the fit needs the drive matrix to be invertible ("This is why the inputs must be at least partially uncorrelated", Smallwood 1999). Method 527.2 asks that the starting signals be "statistically independent and form vectors that are linearly independent with respect to the DOFs to be tested" (4.5.2 b). How low the level is, no source read gives as a rule: Method 527.2 says "very low level" (Annex D 3), AECTP-400 Method 421 calculates "the impedance matrix at low level" and equalizes "up to −3dB" (5.7), and a 2007 maker's data sheet offers −30 to 0 dB relative to the maximum reference rms (Spectral Dynamics 2007).

The **multiple coherence** of a control gauge is the share of its response that the drives explain. Method 527.2 uses it to decide whether H may be updated during the test: "the multiple coherence function will be evaluated to ensure a specific threshold is achieved, (i.e. $\gamma^2 \geq 0.7$). If the user-defined threshold has not been achieved, the transfer function for that spectral line will not be updated" (Annex B 3.4.3). AECTP-400 Method 421 does the same for the correction itself, with no number (5.7). Smallwood (1999) keeps a flag per line for the same purpose. Updating H from the test's own drive works only where the drives are not fully correlated with each other (Method 527.2, Annex B 3.5).

5.2 How Many Averages, and What They Allow

Each loop estimates the response matrix from a finite number of averaged frames, and the estimate scatters about the truth. For Gaussian noise the scatter of one line of an autospectrum depends only on the number of independent averages, and Figure 3 (a) shows it. With 60 averages (120 statistical degrees of freedom, the

number Method 514.8 asks for "If possible"), 90 percent of estimates of a perfectly controlled line fall within −0.98 and +0.87 dB of the truth; with 30, within −1.43 and +1.20 dB. A 2007 maker's figure of 1 dB either way for control "Assumes … 120 DOF, 90% statistical confidence" (Spectral Dynamics 2007); that is about the width of the estimate's own scatter at that number of averages.

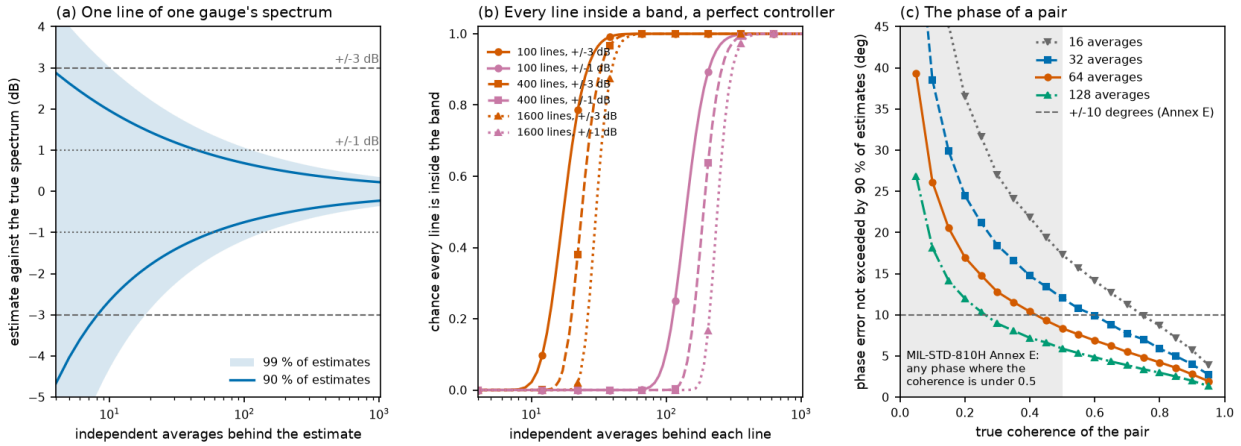


Figure 3: How far an estimate made from a finite number of averages scatters, for a controller that is exactly right. (a) One line of one gauge's spectrum. (b) The chance that every one of 100, 400 or 1,600 independent lines falls inside ±3 dB or ±1 dB. (c) The phase between two gauges, against their true coherence. This article's arithmetic: chi-square quantiles in (a) and (b), a Monte Carlo of 20,000 trials per point in (c).

A tolerance applied to every line at once is harder to meet than it looks, even for a perfect controller (Figure 3 b). With 400 independent lines and 32 averages, every line falls within ±3 dB 89 percent of the time; with 16 averages, 3 percent of the time. Keeping all 400 lines within ±1 dB nine times in ten takes about 260 averages. Method 527.2 expects this: "The portion of the spectrum that actually reaches the maximum tolerance limits is anticipated in narrow bandwidths" (Annex E 5.2 a).

The cross terms scatter more where the coherence is lower (Figure 3 c). At a true coherence of 0.5, 90 percent of phase estimates from 64 averages fall within 8.4 degrees of the truth and from 32 averages within 12.0 degrees, so a ±10-degree phase tolerance (Method 527.2, Annex E 5.2) sits at the edge of what the estimate itself allows. The coherence estimate at a true 0.5 spans 0.40 to 0.60 (5th to 95th percentile) at 64 averages and 0.36 to 0.64 at 32: the Annex's ±0.1 is about the scatter at 64 averages. Annex E also advises a reference coherence "near zero" where the desired coherence is under 0.2, because such estimates are biased, and "If the coherence is small the phase is not important" (4.7). Beyond these remarks, the sources read give two numbers: Method 514.8's 120 degrees of freedom, and AECTP-400 Method 421's statistical degrees of freedom "characterised at the 99% confidence level to achieve a result within 5% of the defined value" (5.1). Neither ties a cross-term tolerance to the number of averages.

More averages mean a slower loop. Each frame lasts one over the line spacing, so at 1 Hz lines 64 frames overlapped by half take about half a minute (this article's arithmetic). Controllers therefore carry averages over from loop to loop (one 2007 data sheet quotes under 0.7 s a loop for four channels and 200 lines with four new frames a loop and 120 degrees of freedom; Spectral Dynamics 2007), and Manring and co-authors' smoothing over lines (Section 4.3) is a way of correcting from fewer frames. The open controller described in Section 9 goes on using the most recent estimate of H if its computation falls behind the output (Rohe 2022).

5.3 Turning a Drive Matrix Into Signals

The drive matrix becomes time signals by factoring it, giving each independent source random phase in the frequency domain, coupling the sources through the factor, transforming each frame to time, and overlapping and adding the frames (Smallwood and Paez 1993, not read here, as Smallwood 1999 cites it). When the loop cannot finish within one output frame, Smallwood (1999) makes new frames from old ones by "Random circular shifts of the columns of the drive matrix … windowing, overlap, and add", time-domain randomization, which "cannot be used if non-Gaussian responses are desired." One detail decides whether the output is stationary: for random frames, the windows' squares must sum to a constant across the overlap, not the windows themselves. A Hann window at 50 percent overlap gives a variance that swings by 3 dB at the frame rate (this article's arithmetic); the open controller's manual applies the square root of the window "which will result in a signal with constant variance" (Rohe, Schultz and Hunter 2024).

6. Limits, Notching and Drive Limits

A single-axis test limits the response at chosen points and notches the drive where it would exceed it. With several shakers the question is which drives to reduce. Method 527.2 allows notching "across all actuators with equal weighting or by weighting notching at each actuator as a function of coherence between the actuators and the location of interest", and limiting on stress or force (Annex E 4.5.4). It notes that while most systems check each exciter's limits, "the feasibility of exciter reproduction relative to cross-spectral information is generally not checked" (4.5.1), and that differences in impedance "may result in excessive drive signals" which makers "address … in varying degrees" (Annex E 5.1.1 d). From single-axis practice it carries the rule that drive clipping should not be used unless needed, and "the limiting threshold should never be set to less than three standard deviations" (Method 514.8, 4.2.2.1). Hunter, Cross and Nelson (2018) add that specifications enveloped with straight lines already drive laboratory force to unrealistic levels at resonances, and that multi-shaker testing "exacerbates this problem".

The makers' pages describe more than they explain: dropping all drives in proportion or "only the drives which will affect the Notch channel", and predictive notching from the pretest (Data Physics, 2026); limit channels that "override defined control method on spectral line by line basis" (Spectral Dynamics 2007); drive clipping at 3 to 10 standard deviations and a maximum drive voltage (Crystal Instruments, 2026). D'Elia and co-authors (2020, abstract) note that excess drive power can stop a test before full level. The open controller's manual says its warning and abort limits work on the autospectra only and "It is not currently possible to set a limit based on, for example, the coherence between two channels" (Rohe, Schultz and Hunter 2024). Underwood wrote on response limiting in multi-shaker tests (2007, as listed in Underwood, Keller and Ayres 2017) and the Nanjing group on a limit strategy (Zheng and co-authors 2023); neither was obtained.

7. Rigs, Shaker Placement and the Points Not Controlled

What a test can do is set largely before the controller starts: by where the shakers and control gauges are. Rohe, Nelson and Schultz (2019) studied a rig with 19 control channels and two validation channels, and 54 candidate shaker locations. The error at the validation channels "reaches a local minimum at 10 or 11 shakers, but then proceeds to grow to a very large value when the problem becomes square", and they concluded that "Even if large shakers are available so that force is not a limiting factor in the test setup, performing square control is not advisable." Schultz and Avitabile (2020) likewise advise "more outputs than inputs". Schultz (2019) found input locations chosen to minimize the condition number gave one lower than a poor choice "by around an order of magnitude over the entire bandwidth". Tuman and co-authors (2024) state the limit plainly, "Given Ns shakers, one can only guarantee that the response will be matched at Ns points", and conclude that "it seems

to be much more important to limit the number of modes active in the transmission simulator for a fixed set of shakers rather than to try to match the flight boundary condition a little more closely."

Method 527.2 says the same from the other side: "no control scheme will force motion of a structure in an unnatural manner" (Annex B 1). The modes of the rig itself set limits too: Paripovic and Mayes (2020) could not hold a fixed-base resonance of the article near 1140 Hz, which "could not be notched enough", nor a twisting mode of the table near 1900 Hz. Schultz and Rohe (2026) note that controlling the table's motion at a virtual point assumes a rigid path from that point to the table's gauges. The first article in this journal measures, in simulation, what happens at held-out gauges when the control gauges are matched exactly (Cross and Zwink 2026).

8. Structures That Change With Level

A bolted structure's resonances move and its damping changes as the level rises. On a structure joined by one bolt, driven by one shaker with band-limited random from 0.14 to 18 lbf rms, Khan and co-authors (2021) report an axial mode near 1250 Hz moving down toward a bending mode near 1130 Hz, about 10 percent, with the damping first rising and then falling (read in their presentation slides). Every law in Section 4 uses an H measured at a low level.

What the multi-shaker random literature read here says about this is brief. Smallwood (1999), in the section on waveform control, writes that the frequency responses are "a linear least squares fit. The fit is usually acceptable for inputs near the excitation level used to measure the frequency response functions", and that "If the system is significantly nonlinear, the full level test may not be acceptable"; in the random section he suggests a nonlinear element between the sources and the drives "to correct for some" of the nonlinearities. Ross and co-authors (2017), deriving inputs rather than controlling them, advise measuring "with inputs as close to the environment that is being replicated as possible." Method 527.2 says of its two assumptions, a linear time-invariant system and a kinematically consistent reference, "Clearly, neither assumption holds for the majority of laboratory vibration tests, even in the SESA case", SESA being one exciter in one axis (Annex B 3.3.2), and that "It is almost always necessary to make an initial run at less than full level" (4.5.2 b), and that the low-level transfer function "may be updated for higher level testing" to account for "certain vibration system amplitude nonlinearities" (Annex D 3). AECTP-400 Method 421 calls on-line modal analysis "essential" to evaluate "non-linear effects caused by the specified vibration and shock operational levels" (2.3.1). The open controller offers to update the identification during control, "useful for nonlinear systems", with two cautions: drives that become too correlated leave the update unable to separate the shakers, and if something in the setup "breaks or comes loose" the controller may keep raising the drive (Rohe, Schultz and Hunter 2024).

No published multi-shaker random test reporting control on a structure measured to change with level was found in the Crossref, OSTI, Semantic Scholar and web searches made in November 2026. The standards and the controllers' documentation say only that the transfer functions are measured at a low level and may be updated as the level rises, and none gives a rule for how far below full level to identify or when to identify again.

9. What Controller Makers and the Open Controller Publish

Makers' public pages and data sheets describe features, not methods, and their figures are their own claims under their stated conditions. One maker's pages say its multi-shaker controller was "Developed Around Mil-810 Method 527", drives 2 to more than 24 shakers, chooses "Over-constrained or square control" by configuration, offers transformations to rigid-body motion and "Mode shape Nulling", and lets the number of controlled degrees of freedom change with frequency (Data Physics, 2026). A 2007 data sheet from another describes an "Adaptive inverse frequency response matrix" with an "Adaptive gain … selectable from 0.0 to 1.0", a singularity threshold for the pseudo-inverse, rectangular control that "produce[s] a form of average control",

and a hybrid of rectangular control at low frequency and square control above (Spectral Dynamics 2007). A third gives up to 512 inputs and 8 shakers, with a weight for each control channel in its multi-exciter single-axis mode (Crystal Instruments, 2026), and a fourth 2 to 4 drive loops controlled as independent, by magnitude and phase, by magnitude only, or as equal outputs (Vibration Research, 2026). Another describes building the reference from the specified autospectra and the cross terms measured during identification, with "equal weighting of these target functions", the drives updated "in a least-squares sense" (Siemens, 2026): the buzz-test reference of Section 3.2 as a maker's practice. Others list multi-shaker random among their products without public method pages that could be read (m+p international, HBK, IMV). Method 527.2 says the tolerance analysis "is usually performed within the MET software" and that understanding it will "In most cases … require direct contact with the vendor" (4.4.3 a).

One controller is open source. Rohe, Schultz and Hunter (2021) describe it: the reference is a cross-power spectral density matrix; the control law is a user's Python function, "generally reasonably short (<20 lines of code)", that "must return an output CPSD matrix from which time histories will be generated", with closed loop available by using the previous drive or response. Its manual and a short-course talk list the laws it supplies as examples: the pseudo-inverse with optional truncation of small singular values, a closed-loop correction to the trace of the specification, the shape constraint of Schultz and Avitabile, and the buzz test "by Daborn" (Rohe, Schultz and Hunter 2024; Rohe 2022). Its application in the 2021 paper is the six-degree-of-freedom table with added modal shakers of Section 1.1, where the talk says a custom law combined the shape constraint and the buzz test. It is described here only from those published documents.

10. Newer and Learning-Based Work

The newest closed-loop work read in full is the convergence study of the matrix power law (Section 4.3), which tuned its settings offline by Bayesian optimization in simulation. A 2021 talk by the same authors proposes reinforcement learning to choose the exponent, with simulations of the plain law and no learning result yet (Manring, Mann and Schultze 2021). The Nanjing group extends multi-shaker random control to references with set kurtosis and to non-stationary references (titles; also a Chinese patent listed as active, Ren, Zheng and Chen 2026). On the reference side, Proner and Mucchi (2024) fix the coherence and phase that give the largest response and solve for the smallest reference spectra that reproduce a target response at a component; on a three-axis shaker with a cantilever and tip mass, the response came within −0.03 to +0.16 dB of target, and the total drive fell 1.06 dB, where a minimum-drives reference reduced it by 0.16 dB.

No neural network or reinforcement-learning controller for multi-shaker random testing with results was found in the searches made for this survey.

11. Where the Published Work Stops

Each statement is worded to the searches and reading behind this survey (Section 12).

1. **No tolerance for points that are not controls.** No standard read sets one, and the published studies that judge tests at held-out points set no pass value.
2. **Few numbers for the cross terms.** Method 527.2′s Annex E gives "a reasonable starting point"; AECTP-400 Method 421 leaves the limits to experiment; IEC 60068-2-86 sets none in its clause 6.
3. **Few printed closed-loop laws.** Smallwood's factor correction is printed in full; the matrix power law is known here through a restatement; makers describe theirs as proprietary or by feature. Manring and co-authors say convergence "has received little attention".
4. **No comparison of closed-loop laws on the same rig** was found. The comparisons read are of references and input estimates (Schultz 2019; Schultz and Avitabile 2020; Musella and co-authors 2018; Proner and Mucchi 2024), or of one law's settings (Manring and co-authors 2023).

5. **The least drive with more than two controls is open.** Method 527.2 says minimum drive techniques "have not been formally established"; Musella and co-authors' fix for three or more channels sets phases by a rule, and they call a general derivation "unfeasible at the authors' knowledge".
6. **Structures that change with level** are not addressed by any published multi-shaker random test found (Section 8), and no published rule was found for the identification level or for when to identify again.
7. **Statistics and tolerances are not tied together.** The standards note that estimation scatter limits how tight a tolerance can be; MIL-STD-810H gives 120 degrees of freedom and AECTP-400 Method 421 a confidence rule, and neither ties a cross-term tolerance to the number of averages.
8. **No learning-based controller with results** was found.

12. Limits of This Survey

The search was made in November 2026 over open sources: OSTI, NASA's technical reports server, Crossref, Semantic Scholar (often rate-limited), patent pages, makers' public pages and general web search. No subscription database was used, Chinese-language databases included. The reading records hold 96 entries: 20 read in full, 32 in part, 10 as abstracts only, 30 not obtained beyond their bibliographic record, and 4 background references recorded only. Sources named in the text but not read are marked as such where they are named. The ones whose content would matter most and could not be had are Cui and co-authors 2011 (the matrix power law in its own words), Smallwood's papers of 1978, 1982 and 1986 and Paez and co-authors 1987, Fisher 1973, Smallwood 2007, Daborn's papers and thesis, Underwood and Keller's papers on rectangular control and transformations, D'Elia and co-authors 2020, the Nanjing group's later papers, the annexes of IEC 60068-2-86, and IEST-RP-DTE022. The open-source controller is described from its published paper, slides and manual only.

This survey describes published work. It does not present, rank or score any control law of its own; laws developed for Hertzforge and the ideas tried and set aside are the subject of separate articles.

How This Was Made

AI tools were used in this work. The literature search, the reading of the sources and the checking of each record, the arithmetic, the figure code and the drafting of the text were done with Claude Code, an AI coding assistant running Anthropic's Claude Opus models, under Kevin Cross's direction. The author is responsible for every word, number and line of code.

Acknowledgments

Much of the published work on multi-shaker random control is the work of Sandia National Laboratories, and it is credited to Sandia National Laboratories: the control algorithms, their lineage and the minimum-drive and independent-drive references of David Smallwood with Paez, Woodall and Buksa; the cross-spectrum study of Hunter and co-authors; the input-estimation, regularization, shape-constraint, placement and test-method studies of Schultz, Avitabile (of the University of Massachusetts Lowell), Rohe, Nelson, Mayes, Paripovic and their co-authors; the six-degree-of-freedom input derivation and test work of Ross, Jacobs and co-authors; and the open-source controller of Rohe, Schultz and Hunter. The convergence work on the matrix power law is that of Manring, Schultze, Zimmerman and Mann, of Duke University and Los Alamos National Laboratory. The matrix power law is the work of Cui, Chen, He and Jiang, and its later extensions that of Cui, Chen, Zheng and their colleagues at Nanjing University of Aeronautics and Astronautics. The minimum-drives target and the drive-power reduction work are those of Musella, D'Elia, Mucchi, Proner and co-authors at the University of Ferrara, Siemens and other universities. The buzz test and the impedance-matched multi-axis test are Daborn's, with Ind, Ewins and Roberts. Intrinsic connection excitation is the work of Zwink and co-authors; the transmission-simulator test is that of Tuman, Allen and co-authors; the Pareto targets are those of Coletti,

Davis (University of Georgia) and Schultz. The industry view is Underwood's, with Keller and Ayres. MIL-STD-810H Method 527.2, IEC 60068-2-86 and AECTP-400 Method 421 are the work of their committees.

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